On the significance of the Braxmaier et al. Laser Interferometry Experiment of 2002 for a Theory of Relativity

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A recent laser interferometry experiment [1] registered a non-detection of a frequency shift that amounts to a fractional change in the speed of light $\delta c/c \approx (4.8 \pm 5.3) \times 10^{-12}$. We comment here on the theoretical significance of this experimental result.

An innovative laser interferometry experiment, wherein the frequencies of two sources were compared over a period of ~ 200 days, used [1] sources of the following characteristics.

An Nd:YAG laser (frequency stabilized by cryogenically cooled, ultra-stable, Nd:YAG laser using the Pound-Drever-Hall method [2]) served as the "length standard" for the interference experiment since the standing wave-mode number, n, of the resonance cavity was held fixed. In this case, the size L of the cavity controls the wavelength of the emitted light as: $\lambda = 2L/n$.

Another Nd:YAG laser (frequency stabilized to a normally forbidden transition of gaseous iodine) served as the "time standard" for the experiment. In this case, the original beam of emission was split into two by a polarizing beam-splitter. The stronger of the parts was then used as the pump beam and the weaker part as a probe beam. The probe beam attenuation by the Iodine gas actively controlled the resonator and stabilized it on resonance frequency with the used Iodine transition.

Thus, if the length-standard's frequency varied independently of that of the time-standard, then the relative frequency shift of these sources would result. So, the $\lambda \approx 1\,\mu$m emission of two sources, one as the length-standard and another as an "independent" time-standard, was allowed [1] to form the interference to measure their frequency difference via the fringe shifts.

This interesting laser interferometry experiment then showed [1] a non-detection of a frequency shift, to be specific, $\delta\nu/\nu \approx (4.8 \pm 5.3) \times 10^{-12}$ over an observation period of ~ 200 days. This is then also the fractional change, $\delta c/c$, in the speed of light, c, indicating that this *constant* of special relativity is unaffected by factors like the motion of the Earth relative to the reference system of the cosmic microwave background radiation [1, 3].

In [3], this result has been shown [15] to mean no or very little expansion of the space, quite contrary to the well-known interpretation of the galactic redshifts and an implication of the standard cosmology [4] based on Einstein's field equations of general relativity. It has thus been used [3] to question the reality of space expansion at the rate inferred from the WMAP data [5].

Of specific interest to us here is the absence of any noticeable effects of the non-inertial character of the Earth's reference system on the frequency shift. Not only is the Earth in motion relative to the cosmic microwave background, it is rotating about an axis as well. The absence of any effects of these motions on the frequency-shift should be of fundamental significance.

Recall that, along with other experiments, the null result of the Michelson-Morley experiment indicated [6, 7] the spurious nature of the "ether." In the same spirit, the non-detection of the frequency-shift in the experiment of [1] indicates that the speed of light (in vacuum) is a *fundamental constant* that is independent of whether the system of reference is inertial or non-inertial.

As was shown by Einstein [6], the constancy of the speed of light (in vacuum) (is inconsistent with the Galilean transformations of Newton's theory but it) *can be made consistent* with the special principle of relativity using the Lorentz transformations, the special principle also being the basis of the Galilean transformations. Similarly, if this constancy is the above fundamental law, as it seems from [1], then it needs to be made consistent with the general principle of relativity by selecting a proper mathematical formulation for this principle [16]. It is but easily seen that Einstein's field equations of the general relativity are not consistent with this *complete* constancy of the speed of light.

Einstein, likewise with Newton, had aimed at a comprehensive theoretical description of all the physical phenomena. Thus, his field equations of the general relativity *describe the physical entirety* on representing "gravity" by the Einstein tensor and on incorporating the "remaining" forces by way of the energy-momentum tensor. Of course, only "gravity" gets represented by the spacetime curvature in these, now famous, Einstein's field equations.

But, to go beyond Newton's theory, the concept of force needs (logical) replacement, and not any specific type of force like gravity. It then follows that Einstein's equations are [8] logically unacceptable. The solutions of Einstein's field equations are then inappropriate for describing any physical situations. That is to say, various (non-Newtonian) properties of the solutions of Einstein's field equations should be irrelevant or inconsequential to any of the physical situations.

This above conclusion is inescapable even when Einstein's field equations provide, as solutions, 4-dimensional (spacetime) geometries, mathematical constructions, which only are mathematically well-defined but physically certainly dubious.

In fact, by 1928, Einstein had concluded [9, 10] that his field equations were not any satisfactory formulation of the general principle of relativity. He had thus discarded these field equations, being physically ill-posed (see later). Importantly, these are also logically unacceptable [8].

Certain cosmological solutions [4] of Einstein's equations imply an "expanding" space. In view of the logical unacceptability of these equations, it is then not surprising that the result of the Braxmaier et al. experiment [1] is inconsistent [3] with the implied expansion of the space.

Now, the general principle of relativity "looks" for *all* the laws of physics that are the *same* for *all* the observers, undergoing whatsoever relative motions. Then, these laws of physics will also include the laws of the quantum. This is the reason why we can expect that a suitable theory based on this principle of relativity fundamentally includes the phenomena also from the quantum world.

Clearly, the general principle of relativity then seeks also for *all* the laws about *all* the permissible interactions of physical bodies.

The general principle of relativity is a basis [11] for the unification of interactions then. It is a basis provided that we develop a suitable mathematical framework for it.

In this connection, Einstein had expressed [12] his valued judgement about the general principle of relativity that the idea of general relativity "is a purely formal point of view and not a definite hypothesis about nature ...".

What he means here is that the general principle of relativity is not a statement about the (character of the laws of) Nature, but it is rather that this is a statement about how one should, as a strategy, obtain the laws of Nature.

Then, rather than restricting ourselves to the confines of the special principle of relativity, it is "advisable" to formulate a theory on the basis of the general principle of relativity. This should get viewed indeed as a statement about a definite strategy on as to how to obtain the laws of Nature, and not as a hypothesis about Nature.

Within this strategy of the general principle of relativity, it is then not pivotal as to how many species of (elementary) particles exist; what their specific properties as well as interrelationships are, how many fundamental forces Nature has, etc.

As long as we have a satisfactory mathematical framework for the general principle of relativity, these above (and results alike) could then be looked upon as some "predictions" of the corresponding theoretical framework. Essentially, a theory (as a conceptual and the corresponding mathematical framework) will have to tell us what its "observables" are.

Moreover, such predictions of a "theory of everything" will be "verifiable" and could then establish or demolish the chosen mathematical premise of the general principle of relativity, but not the principle, which is a purely formal point of view. It therefore follows that if one mathematical formalism fails to explain observations, then we replace it by suitable other.

This is the meaning of the general principle of relativity being a purely formal point of view then. This is also why Einstein had been searching for a suitable mathematical framework to represent this principle, and why he had expressed some doubts [9] [p. 8] as to whether the standard differential geometry would help with further progress. (See later.)

It is well known [9, 10] that Einstein attempted numerous formulations for what he called the unified field theory. All these formulations were however unsatisfactory even for their creator. These later years of his scientific career often get described [9] [p. 341] as: "he was as clear about his aims as he was in the dark about the methods by which to achieve them."

To understand the problem that so persistently troubled Einstein, let us represent a material body as a point of the 4-dimensional spacetime geometry. This we do because Newton's theory so represents a physical body, explains some phenomena and we demand that our field equations reduce to the appropriate equations of Newton's theory in some suitable approximation. Moreover, it is also imperative that any theory of everything resolves this problem satisfactorily.

Now, let a point that is a curvature singularity of the geometry represent a physical body as in, *eg*, Schwarzschild's famous solution of Einstein's equations. Any mathematical expressions valid at other points of geometry do not hold at its curvature singularity. Then, the "motion" of a curvature singularity is a "singular" curve and it is not part of that geometry. Therefore, equations for a curve of the geometry, geodesic equations, do not provide a law of motion for the singularity. In the absence of any law of motion for point objects, it is not possible to define their fluxes and, hence, the energy-momentum tensor. In this case, Einstein's equations do not have any physical meaning whatsoever. Einstein had recognized this problem of the "pure gravitational fields" as the problem of locations in space where his field equations of general relativity are not valid [9, 10].

Next, let a chosen point of geometry be not its any curvature singularity. Now, geometric equations (geodesic, geodesic deviation) acquire *physical meaning* only when any point of geometry at which they are satisfied is ascribed mass, charge etc. This is indeed the same as Newton's theory using Euclidean geometry and ascribing mass, charge to a point on the curve of that geometry with equations of the curve providing Newton's laws of motion. In the case of Einstein's equations, only the geometry is non-Euclidean. This is then the only relevant difference in the two cases.

Now, an unambiguous way to ascribe mass etc. to any point of the geometry is needed to define the energy-momentum tensor. In Newton's theory (Special Theory of Relativity), this prescription is "by hand" for any point of the Euclidean (Minkowskian) geometry. How then for Einstein's case of a curved 4-dimensional geometry is this unambiguously done?

Following Newton, we may prescribe, by hand, values of mass, charge etc. for any non-singular point on the curve of a (curved) four-dimensional geometry, formally define the energy-momentum tensor, write Einstein's field equations and obtain their (spacetime) solutions.

Where is the problem then? The problem is that the mass of a physical body is represented by the curvature of the spacetime geometry. "More the matter, the larger is the curvature." This leads [13] to the problem of defining the energy (or mass) of a system in general relativity. As is well known [13], there is no satisfactory definition of mass in general relativity and, hence, the above prescription of mass for a point of curved geometry cannot be made in any unambiguous manner. Hence, mass cannot be the part of the energy-momentum tensor in the above simple way.

By comparison to this trouble of Einstein's field equations, Newton's theory faces no such difficulty. This is so because mass, charge etc. are the "extraneous" mathematical quantities, which are independent of the nature of the Euclidean geometry underlying Newton's theory. These characteristics can, therefore, be specified, by hand and at will, for a point of that geometry.

But, unambiguous prescription of mass, charge etc. for a point of the curved geometry becomes *impossible* when we attribute the strength of the gravitational field to the curvature of the (spacetime) geometry. Einstein expressed [10] this difficulty in the words that the "right side" of his equations is "a formal condensation of all things whose comprehension in the sense of a field theory is still problematic." That is to say, his field equations are physically ill-posed.

We also note that a curved (spacetime) geometry is not the unique mathematical notion implied by Einstein's equivalence principle, or by the general principle of relativity; the latter being the actual basis of general relativity. In fact, the general principle of relativity does not "prescribe" any specific mathematical framework for itself.

This "explains" why Einstein explored [9] the question of whether the fundamental mathematical basis for physics might be other than that of the partial differential equations and had expressed [9] [p. 8] doubts as to whether differential geometry be used for further progress [17].

The pivotal issue here is then of an appropriate mathematical framework for the ideas behind the general principle of relativity. The following then appears to be a relevant step in the direction of this mathematical framework.

Historically, Descartes [7] was the first to express concern that a coordinate system of the Euclidean space of the Newtonian Mechanics, as a "material construction" of a reference body, does not get affected by physical precesses. Newton recognized this in the form of the "absolute space" underlying his Mechanics. Systems of reference in Newton's theory and in special relativity, both, cannot be any material bodies then. This is a definite drawback of these basic theories.

Unless we address Descartes's above issue, no mathematical framework for the general relativity would be satisfactory. Hence, the *general* principle of relativity must deal with the "physical" reference systems, physical bodies, undergoing whatever relative motions. This principle of relativity can therefore find its proper mathematical representation only when we have an appropriate mathematical structure to represent all the physical bodies usable as reference systems.

Changes in the physical bodies are the *physical phenomena*. As only physical bodies get used as reference in physical measurements, mathematical structure(s) representing physical bodies will have to be such that phenomena become 'changes' to (transformations [18] of mathematical structure(s) of) reference systems themselves.

Then, a mathematical framework, which is different than that of the standard differential geometry, that properly represents physical bodies and that is therefore consistent with the general principle of relativity is needed [14] for further progress. This mathematical framework needs to be based on the concept of a Quasi-Category [11] and, as such, was called the Universal Relativity to differentiate it from Einstein's general relativity, both being based on the general principle of relativity.

As discussed here, the result of the Braxmaier et al experiment [1] appears to broadly support (certain parts of) Einstein's viewpoints of later years [9, 10] and also the general approach together with the mathematical methodology in [11, 14], both. This approach that utilizes the category theoretical methods to represent the general principle of relativity is then a promising program for the unification of all the fundamental physical interactions.

We therefore conclude that the Braxmaier et al experiment [1] rules out Einstein's field equations but supports the general principle of relativity, a strategy for developing a theory.

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[15] A measured value was $\delta\nu \sim 1.36 \pm 0.63$ kHz, while that implied by the (flat) standard model of cosmology is $\delta\nu \sim 6$ kHz. One could, however, argue that the cosmological expansion has already "decoupled" from the locally measurable effects.

[16] Point transformations of coordinates could be used to implement the constancy of the speed of light. However, such a formalism will not be satisfactory in view of Descartes's concern discussed later.

[17] Within the mathematical framework of Standard Differential Geometry, "values" of mass, charge etc. can only get "specified by hand" for a point of the geometry. Mathematical methods away from differential geometry are, therefore, needed for appropriately "representing origins" of the physical conceptions of inertia etc.

[18] Dirac [(1939) *Proc R Soc (Edinb)*, **59**, 122] had emphasized the importance of transformations as: "... transformations play an important role in modern physics; both relativity and quantum theory seeming to show that transformations are of more fundamental importance than equations."